Determining the accurate object distances is always important in simple photogrammetry. In the photographs of mountainous area, however, the determination is a difficult problem. In order to solve this problem, the following equation is proposed for the photo scale number $m_B$ at the lower elevation of the two end points of the ground line:

$$m_B = \left[ \sqrt{(s_h \cdot m_s)^2 - (v \cdot \Delta h/f)^2} + d \cdot \Delta h/f \right] / s_h$$

With this $m_B$, accurate object distances can be computed. To keep the accuracy better than 1/100, $s_h \cdot m_s \geq 7v \cdot \Delta h/f$ is defined as the necessary condition.

The methodology of simple photogrammetry with non-metric photographs is described in this paper. The method of taking horizontal-normal photographs and necessary conditions for three dimensional measurement in simple close-range photogrammetry are also discussed.

The results of measurement by parallax-meter are compared with the results of measurement by analytical plotter PA 2000. The results of PA 2000 are adjusted by the bundle method with self-calibration.

This comparison proves that the parallax-meter measurement is effective. In addition, this paper describes the several points to be ameliorated about parallax-meters and spiral parallax measuring boards.

1. Introduction

Mirror stereoscopes and parallax-bars are commonly used as the student practice materials. As they are relatively expensive, one instrument cannot usually be allotted to each student. To overcome the expense, one of the authors designed a simple parallax-meter with moderate accuracy. This device can be handled easily, and it can also be purchased by individual students. The automatic contouring technique by means of stereo-matching has been recently studied in the laboratory. However almost all actual mapping works are still done by using analog plotters, so it is important for the students to master the technique of setting floating mark on the model exactly with practice of parallax-meters.

This paper describes test works of simple measurements with parallax-meters using aerial photographs and non-metric hand camera photographs.
2. Accurate Equations for Determining Absolute Flying Heights

Accurately determining absolute flying heights is essential in simple airphoto measurements. Dr. P. R. Wolf in the textbook “Elements of Photogrammetry” describes the flying height $H$ in the following manner:

\[
(AB)^2 = (X_B - X_A)^2 + (Y_B - Y_A)^2
\]

\[
(AB)^2 = \left[x_B \frac{(H - h_B)}{f} - x_A \frac{(H - h_A)}{f}\right]^2 + \left[y_B \frac{(H - h_B)}{f} - y_A \frac{(H - h_A)}{f}\right]^2
\]

where, $X_A$, $X_B$ and $Y_A$, $Y_B$ : ground coordinates of points A and B.

$x_A$, $y_A$ and $x_B$, $y_B$ : photographic coordinates of points A and B.

$h_A$ and $h_B$ : elevations of points A and B.

$H$ : flying height above sea level (absolute flying height).

Since this equation is difficult to apply in practice, the author had deduced the following simpler equation to obtain the elevation $h_c$ corresponding to $m_B$ which is determined by using a sloping ground line:

\[
h_c = h_B + d \cdot \Delta h / s_b
\]

where, $s_b = \overline{ab}$ and $d$ is the distance between $t$ (perpendicular points to line $ab$ from principal point $p$) and $a$ (the higher point of A and B on the photograph) and $\Delta h$ is the elevation difference : $\Delta h = h_A - h_B$.

The photo scale number $m_B$ at the elevation of the lower point B can be derived from the following equation:

\[
m_B = (s_k \cdot m_k + d \cdot \Delta h / f) / s_b
\]

The distance between two points is required to satisfy the condition. $s_k \cdot m_k \geq 7 \cdot \Delta h$

$m_B$ : photo scale number at the elevation of the lower point B.

$m_k$ : map scale number.

$s_k$ : distance of two points on the map.

$f$ : camera focal length.

Because satisfying the above conditional equation in large-scale mountain photographs is difficult, a more exact approach and stringent checking must be used (Fig. 1). By the theorem of three perpendicular lines

\[
\angle Ota = \angle AA' = 90^\circ
\]

Consequently,

\[
\Delta Ota \sim \Delta AA', \Delta Opt \sim \Delta AA'A''
\]
From these similar triangles, the most accurate and practical equation for determining the photo scale number \( m_B \) is derived as follows:

\[
m_B = \left[ \sqrt{(s_k \cdot m_k)^2 - (v \cdot \Delta h / f)^2} + \frac{d \cdot \Delta h}{s_b} \right] / s_b \tag{3}
\]

where, \( v \) is perpendicular length to the line \( ab \) from principal point. As long as the following limitation \( s_k \cdot m_k \geq 7v \cdot \Delta h / f \) is satisfied, then the term of \( (v \cdot \Delta h / f)^2 \) is negligible and \( m_B \) maintains the relative accuracy higher than 1/100. The accurate absolute flying height \( H_0 \) is consequently obtained from the equation:

\[
H_0 = m_B \cdot f + h_b
\]

3. Some Examples of Determining Absolute Flying Height by Equation (3)

Photographs used for this test are photo-number No. 4 and No. 5 of course C1 of Yamakawa district mission in Shikoku. The orientation elements by analytical aerial triangulation are as follows:

\[
\begin{align*}
\kappa_1 &= 105.73 \text{ gon}, \quad \phi_1 = 103.218 \text{ gon}, \quad \omega_1 = 99.114 \text{ gon} \quad \text{in photo No. 4} \\
\kappa_2 &= 105.62 \text{ gon}, \quad \phi_2 = 101.020 \text{ gon}, \quad \omega_2 = 99.178 \text{ gon} \quad \text{in photo No. 5} \\
b_x &= 159.375 \text{ mm}, \quad b_y = 100 \text{ mm}, \quad b_z = 96.729 \text{ mm}, \quad f_1 = 152.66 \text{ mm}, \quad f_2 = 152.66 \text{ mm}
\end{align*}
\]

In Fig. 2-1 and Fig. 2-2, the number on the ground line shows the absolute flying height determined from the ground line. In Fig. 2-1, all absolute flying heights are approximately equal, because No. 5 photograph is kept in the limit of 1 gon in both \( \phi \) and \( \omega \).

In photo No. 4, because \( \phi \) is more than 3 gon, the absolute flying heights show the quite big differences and the ground lines near the diagonals should be selected.

Finally, absolute flying heights are determined as 3,480 m and 3,507 m for No. 4 and No. 5, respectively. These determined heights agree well with the results of analytical aerial triangulation.
Kiyoo SAZANAMI and Mitsuharu YAMADA

Fig. 3 Absolute flying heights (unit: m) corresponding to each ground line and the frame of photo (C15-5), (Gifu district). The broken line designates the boundary of the stereo-photo (Fig. 6).

In another trial aerial photograph No. 5 in Gifu district was used. The ground lines with large elevation differences were compared to those with small elevation differences in this trial. As long as equation (3) is used, the same results are obtained. (Fig. 3)

4. Simple Measurement of Aerial Photographs

4.1 Measurement of Large-Scale Mountain Photographs

For height measurement, a parallax wedge of the ladder type (trade name: parallax-meter) and a spiral parallax measuring board which the author has recently developed were used. See Fig. 4 and Fig. 5. The former had the measurement accuracy of ±(0.05~0.03) mm; the latter, that of ±(0.05~0.02) mm.

According to the usual height formulae, \( \Delta H = H \cdot \Delta P / b \), \( \Delta H = H \cdot \Delta P / (b + \Delta P) \), where \( H \) is the flying height above the object ground. It is also a variable. Therefore a series of orderly A, B, C, D and E measurements must be made by using these formulae:

\[
\begin{align*}
\Delta H_{AB} &= H_0 \cdot \Delta P_{AB} / P_A, \\
\Delta H_{CD} &= (H_n + \Delta H_{BC}) \cdot \Delta P_{CD} / P_D,
\end{align*}
\]

\[
\begin{align*}
\Delta H_{BC} &= H_0 \cdot \Delta P_{BC} / P_C, \\
\Delta H_{DE} &= (H_c + \Delta H_{CD}) \cdot \Delta P_{DE} / P_E.
\end{align*}
\]
In simple close-range photogrammetry, these formulae are very well matched. For these measurements, the direct measurement of the parallax by the parallax-meter is superior to the parallax bar. Parallax bar measurement requires the parallax corrections between principal points and object points. Also the measuring marks are likely to move back and forth.

The parallax-meter, on the other hand, measures stably. It can be operated quickly and easily. In addition, it improves the reliability of measurements by comparing adjacent floating marks. The spiral parallax measuring board which has gentle slopes of floating marks offers superior approaches to measurement.

4.2 Preparation of Prallax Correction Graph

Using stereo-photo in Fig. 6, the parallax correction graph was made. As shown in Fig.6, about 20 points were selected in the overlapping area and the elevations of the points were taken from the topographical map. The lowest point, No.5, was selected for the datum and parallax differences, \( \Delta P_{1-5} \), \( \Delta P_{2-5} \), the others were calculated from the following formulae,

\[
\Delta P_{1-5} = P_5 \cdot H_{1-5} / H_1, \quad \Delta P_{2-5} = P_5 \cdot H_{2-5} / H_2, \ldots.
\]

The parallax \( P_5 \) of point No.5, however, was measured by a parallax-meter. The flying heights above ground: \( H_1 = H_0 - h_1, H_2 = H_0 - h_2 \cdot \cdot \cdot \), elevation differences: \( \Delta H_{1-5} = h_1 - h_5, \Delta H_{2-5} = h_2 - h_5 \cdot \cdot \cdot \), and the others were all known values. Next, actual parallax differences \( \Delta P_{1-5} \), \( \Delta P_{2-5} \) and others were measured by a parallax-meter. Then \( v = \Delta P - \Delta P' \) is a correction of parallax difference.

The absolute flying height \( H_0 = 3,897 \text{ m} \); the distance between the two principal points \( L = 144.5 \text{ mm} \); the reading values of the parallax-meter at No.5 point \( l_5 = 66.50 \text{ mm} \) and \( P_5 = L - l_5 = 78.0 \text{ mm} \) were all obtained. Table 1 shows the results of measurement and calculation. Fig.7 shows the parallax correction graph.
Fig. 6 The stereo-photograph of Gifu district which were taken by a RC 20, May 17, 1990. The flight line is oriented north-south.

Table 1 Parallax Corrections.

<table>
<thead>
<tr>
<th>Control point</th>
<th>Elevation (m)</th>
<th>Measured $\Delta P$ (mm)</th>
<th>Calculated $\Delta P_c$ (mm)</th>
<th>Correction $v = \Delta P - \Delta P_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>18</td>
<td>0.02</td>
<td>-0.15</td>
<td>0.17</td>
</tr>
<tr>
<td>2</td>
<td>18</td>
<td>0.02</td>
<td>-0.10</td>
<td>0.12</td>
</tr>
<tr>
<td>3</td>
<td>19.5</td>
<td>0.05</td>
<td>0.10</td>
<td>-0.05</td>
</tr>
<tr>
<td>4</td>
<td>28</td>
<td>0.22</td>
<td>1.20</td>
<td>-0.98</td>
</tr>
<tr>
<td>5</td>
<td>17</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>18</td>
<td>0.02</td>
<td>0.10</td>
<td>-0.08</td>
</tr>
<tr>
<td>7</td>
<td>21</td>
<td>0.08</td>
<td>0.25</td>
<td>-0.17</td>
</tr>
<tr>
<td>8</td>
<td>251</td>
<td>5.01</td>
<td>5.40</td>
<td>-0.39</td>
</tr>
<tr>
<td>9</td>
<td>36</td>
<td>0.38</td>
<td>0.70</td>
<td>-0.32</td>
</tr>
<tr>
<td>10</td>
<td>282</td>
<td>5.72</td>
<td>6.40</td>
<td>-0.68</td>
</tr>
<tr>
<td>11</td>
<td>29</td>
<td>0.24</td>
<td>0.95</td>
<td>-0.71</td>
</tr>
<tr>
<td>12</td>
<td>178</td>
<td>3.38</td>
<td>4.30</td>
<td>-0.92</td>
</tr>
<tr>
<td>13</td>
<td>180</td>
<td>3.42</td>
<td>3.90</td>
<td>-0.48</td>
</tr>
<tr>
<td>14</td>
<td>31</td>
<td>0.28</td>
<td>0.80</td>
<td>-0.52</td>
</tr>
<tr>
<td>15</td>
<td>30</td>
<td>0.26</td>
<td>0.60</td>
<td>-0.34</td>
</tr>
<tr>
<td>16</td>
<td>22</td>
<td>0.10</td>
<td>0.75</td>
<td>-0.75</td>
</tr>
<tr>
<td>17</td>
<td>48</td>
<td>0.63</td>
<td>1.10</td>
<td>-0.47</td>
</tr>
<tr>
<td>18</td>
<td>63</td>
<td>0.94</td>
<td>1.40</td>
<td>-0.46</td>
</tr>
<tr>
<td>19</td>
<td>100</td>
<td>2.98</td>
<td>3.35</td>
<td>-0.37</td>
</tr>
<tr>
<td>20</td>
<td>33</td>
<td>0.32</td>
<td>0.45</td>
<td>-0.13</td>
</tr>
</tbody>
</table>

Fig. 7 Parallax correction graph. (unit : mm)

① : Vertical control point.
(-0.6) : Interpolated value.

(6)
5. Three-Dimensional Measurement of Non-Metric Photographs

5.1 Parallax-Meter Method

To use the parallax-meter in non-metric photos, it is necessary to keep the camera axis exactly in the horizontal plane and normal to the base line. This maintenance is accomplished by sticking tape on the objects at the same height as the camera. This tape should then coincide with the central lateral line of the camera viewfinder. This coincidence maintains the horizontality of the photo-camera axis. In normal photography, the problem of convergent and divergent camera axes may be avoided by using two targets and by pacing off the base $B$ measurement. Marking the diagonal intersection of a photograph with a needle to know the deviation from the exact horizontal and normal photography is a good practice as seen in Fig. 8.

Before the actual photograph is taken, an article whose size is known should be placed alongside the object. A book or a pack of cigarettes would a usable article, but longer distances from the object would dictate larger articles.

This article can then be replaced as control points. The article itself should be parallel to the camera axis, not oblique to it.

It should be placed near the lower part of the object. If the position of the article is higher than the camera, the depth of the article cannot be photographed.

Unlike those of aerial photographs, the photo bases of close-range stereo-photography are generally
very short. Principal points do not have to be transferred to coincide on the same base line. As shown in Fig. 8 orientation can be easily accomplished by using objects with horizontal lines like a blackboard and by keeping two corresponding points about 60 mm apart.

Fig. 9 shows the test targets taken by a non-metric camera (lens: NIKKOR f = 50 mm). Ground coordinates of the intersection points of tiles were maintained within the accuracy of ± 10 mm. Object distances were regarded as unknown. They were found by using the depth of the known article. In the formula \( Y = P \cdot \Delta Y/\Delta P \), the depth \( \Delta Y \) is known. Then, \( Y \) is determined by measuring \( P \) and \( \Delta P \) with a parallax-meter. The same equations as those used in large-scale mountain photographs (Section 4.1) can be used. Fig. 10 shows the discrepancies between \( Y \) coordinates obtained on the basis of the given distance \( \Delta Y \) (A17~A37) and the ground \( Y \) coordinates. The accuracy of A2-, A3-, A4- points is rather good. The accuracy of the forward depth can be improved by increasing the known \( \Delta Y \) distances on both sides.

Fig. 11 shows the errors of \( Y \) coordinates in the case of the given distance \( \Delta Y \) (A17~A87). These results have proved to be fairly effective. That means the known base line length must be given on the photograph and it can be defined as the line of the same depth as the object.

### Fig. 10
Discrepancies between \( Y \)-coordinates calculated in the case of the given depth \( \Delta Y \) (A17~A37) and ground \( Y \)-coordinates. Non-metric photographs (f = 50 mm) were measured with the parallax-meter. (unit: mm)

### Fig. 11
Discrepancies between \( Y \)-coordinates calculated in the case of the given depth \( \Delta Y \) (A17~A87) and ground \( Y \)-coordinates. Non-metric photographs (f = 50 mm) were measured with the parallax-meter. (unit: mm)
Fig. 12 Photogrammetric scale consists of both large and small wedge. (unit: mm)

Fig. 13 The stereo-photo of test targets taken by the non-metric camera (lens: SIGMA MINIWIDE $f = 28$ mm)

Fig. 14 Discrepancies between $Y$-coordinates calculated in the case of the given depth $JY$ (A17~A127) and ground $Y$-coordinates. Non-metric photographs ($f = 28$ mm) were measured with the parallax-meter. (unit: mm)
To find the length $\Delta X$ (lateral direction), the principal distance $c$ for the enlarged photo with service size must be calculated. The measured ground distance $\Delta X$ (A16~A17) was 150 mm. The distance $\Delta x$ on the photo was 14.1 mm which was measured with the large wedge scale of the Photogrammetric Scale (Fig. 12). Here $c = 202.7$ mm can be calculated from $\Delta x / \Delta X = c / Y (0 \sim 17)$. The term $Y (0 \sim 17)$ in this equation is the distance from projection center (exposure station) to the point A17. Then $\Delta X$ of the points A46~A47 was given as 150 mm by using the equation $Y (0 \sim 47) = Y (0 \sim 17) + Y (A17 \sim A47)$. $\Delta x = 10.0$ mm and $c = 202.7$ mm. The error was 1 mm in this case.

For the large wedge scale, using stereoscope of 4 magnification is better. For a small object less than 5 mm, a small wedge scale. This scale needs a 15 x magnifying glass. Fig. 13 shows the stereophoto of test targets taken with super wide angle lens ($f=28$ mm SIGMA MINI WIDE) and Fig. 14 shows the discrepancies between $Y$ coordinates obtained on the basis of the given distance $\Delta Y (A17 \sim A127)$ and the ground $Y$ coordinates.

### 5.2 Results of Analytical Plotter PA-2000

The correction for camera lens distortion was not applied, because inner orientation elements are usually unknown. The points A24, A26, A29, A42, A44, A49, A64, A66, A69 were used as the control points for the absolute orientation.

The standard deviations of coordinates derived from measurements of 91 points were $\sigma_x = \pm 36.0$ mm, $\sigma_y = \pm 45.2$ mm, $\sigma_z = \pm 13.9$ mm in $f=50$ mm normal camera.

The standard deviations of coordinates derived from measurements of 132 points were $\sigma_x = \pm 31.9$ mm, $\sigma_y = \pm 45.2$ mm, $\sigma_z = \pm 43.9$ mm in $f=28$ mm super wide angle camera.

Fig. 15 shows the discrepancies of $Y$ coordinates in $f=50$ mm normal camera and Fig. 16 shows

![Fig. 15 Discrepancies between $Y$-coordinates obtained by analytical plotter PA-2000 and ground $Y$-coordinates. ($f=50$ mm). (unit : mm)](image1)

![Fig. 16 Discrepancies between $Y$-coordinates obtained by analytical plotter PA-2000 and ground $Y$-coordinates. ($f=28$ mm). (unit : mm)](image2)
the discrepancies of $Y$ coordinates in $f = 28$ mm super wide angle camera.

### 5.3 Results of Analytical Method

The results by the bundle method with self-calibration (camera: $f = 28$ mm) are as follows: (unit: mm) (unit: gon)

<table>
<thead>
<tr>
<th>The coordinates of central projection points</th>
<th>$X$</th>
<th>$Y$</th>
<th>$Z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Left photo</td>
<td>757.44</td>
<td>-2009.40</td>
<td>568.28</td>
</tr>
<tr>
<td>Right photo</td>
<td>1066.39</td>
<td>-2003.37</td>
<td>570.61</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>The orientation elements of camera</th>
<th>$\kappa$</th>
<th>$\psi$</th>
<th>$\omega$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Left photo</td>
<td>-0.418</td>
<td>1.256</td>
<td>-4.928</td>
</tr>
<tr>
<td>Right photo</td>
<td>-0.237</td>
<td>1.127</td>
<td>-5.023</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>The inner orientation elements</th>
<th>$X$</th>
<th>$Y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>The displacements of virtual principal points</td>
<td>$X$</td>
<td>$Y$</td>
</tr>
<tr>
<td>Left photo</td>
<td>-0.121</td>
<td>-0.001</td>
</tr>
<tr>
<td>Right photo</td>
<td>0.153</td>
<td>-0.003</td>
</tr>
</tbody>
</table>

### 6. Conclusion

In parallax wedge measurement, measurement accuracy is determined by the minimum measuring mark interval, because interpolating a fraction of adjacent measuring marks is difficult. The author proposes that both side lines of the parallax-meter should be changed to zigzag lines with 1 cm pitch. The right side zigzag lines should be angled at 45° to the horizontal line. The minimum measuring mark interval of both parallax-meter and spiral parallax measuring board should be also changed to 0.02 mm.

The authors express the heartful thanks to Asia Aerial Survey Co. for the analytical works by bundle method with self-calibration.

The authors also wish to thank Professor Oshima of Hosei University who made many valuable suggestions.

### References


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